# From Oberwolfach, with Love

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In this talk, we shall explore particular types of problems in cycle decompositions of graphs — a vibrant area of research in the intersection of graph theory and the theory of combinatorial designs. In the last few years alone, several long-outstanding conjectures in this area have been proved, in addition to many other new results. However, many pertinent problems remain open, and all these powerful new results can now serve as tools to tackle problems that were previously inaccessible.

Our story begins with the following problem proposed by Reverent T.P. Kirkman in an 1850 issue of *The Lady's and Gentleman's Diary*:

"Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk twice abreast."

This puzzle is likely the first recorded scheduling problem that can be solved using cycle decompositions of graphs. In this talk, we shall focus on a generalization of the Kirkman Schoolgirl Problem, namely, the Oberwolfach Problem, which was formulated in 1967 by Gerhard Ringel as follows:

"At a conference in Oberwolfach, $2k+1$ participants are to be seated at $t$ round tables for $k$ meals so that each participant sits next to every other participant at exactly one meal. Can this be achieved with tables of sizes $m_1, m_2, \ldots, m_t$ if $m_1 + m_2 + \ldots + m_t = 2k+1$?"

This basic variant of the Oberwolfach Problem can be modeled as a decomposition of the complete graph on $2k+1$ vertices into 2-factors, each of these 2-factors consisting of $t$ disjoint cycles of lengths $m_1, m_2, \ldots, m_t$.

Over the last few decades, many cases of the Oberwolfach Problem have been solved, however, the general problem is still open. In this talk, I will give an overview of the most prominent solved cases, and then focus on the variants of the problem that I have been involved with: the directed Oberwolfach Problem, the Spouse-Loving Variant (or the minimum covering variant), and the Honeymoon Oberwolfach Problem.

No previous knowledge of graph theory will be required to understand this talk.